differences on assumptions / requirements on geometriies for ito framework vs stratonovich framework. I think ito is impliciitly Euclidean

Yes, you're correct that the Itô and Stratonovich stochastic calculus frameworks differ significantly in their geometric assumptions and compatibility with differential geometry.

1. Coordinate Dependence and Geometry:

- **Itô calculus** assumes a **Euclidean structure**. The differential rules (especially Itô's lemma) rely on coordinates and do not transform covariantly under changes of variables. This means:
  - Itô integrals are **not invariant** under nonlinear changes of coordinates.
  - The Itô correction term (second-order derivative term) emerges due to the quadratic variation, which lacks a natural interpretation in general manifolds.
- **Stratonovich calculus**, in contrast:
  - Is **coordinate-free** and **geometrically natural**.
  - Compatible with the chain rule from standard calculus.
  - Integrals transform covariantly under smooth changes of coordinates, making it well-suited for applications on **Riemannian manifolds or more general smooth manifolds**.

2. Requirements on Geometry:

- **Itô Framework**:

- Requires a **flat Euclidean space** structure (implicitly assumes $\mathbb{R}^n$ with a fixed global coordinate system).
- No need for a connection or manifold structure, but if extended to a manifold, it requires defining a connection that incorporates the Itô correction—non-trivial and unnatural.

- **Stratonovich Framework**:
  - Naturally extends to **smooth manifolds**, where vector fields and stochastic flows can be defined intrinsically.
  - Makes use of standard differential geometric tools like **connections**, **parallel transport**, and **Lie brackets** without modification.
  - Allows defining SDEs using vector fields on manifolds directly: $dX_t = V(X_t) \circ dB_t$ makes geometric sense.

3. Implications for Modeling:

- Use **Itô** if:
  - Working in fixed Euclidean coordinates.
  - Interested in martingale properties or expectations (e.g., financial applications).
  - Analytic, probabilistic structure is primary.
- Use **Stratonovich** if:
  - The system evolves on a manifold (e.g., rotation groups, Lie groups, surfaces).
  - Interested in **geometric or physical modeling**, particularly where the SDE arises as a limit of smooth deterministic systems (e.g., Langevin dynamics).

Preferred framework for geometric settings: **Stratonovich**.