Blue Mountain
CAPITAL MANAGEMENT LLC

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Variance and Convexity: A Practitioner's Approach

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Part 1: Variance Risk Exposures and Relationship to Options

Why Variance and not Volatility?

The PnL from an options position is driven by realized variance, not volatility

$$50\sum_i \Gamma_i \left[R_i^2 - \sigma_I^2 \frac{1}{252} \right]$$

where:

Γ_i is dollar gamma on day i

R_i is the return on the underlying on day i

σ_I is the traded implied volatility

Trading a variance future is a way to take a clearly-defined view on realized variance

$$N \times \left(\sigma_R^2 - K_0^2 \right)$$

where:

N is varaince units

σ_R is realized volatility

K_0 is the traded variance strike

Variance Greeks

Vega

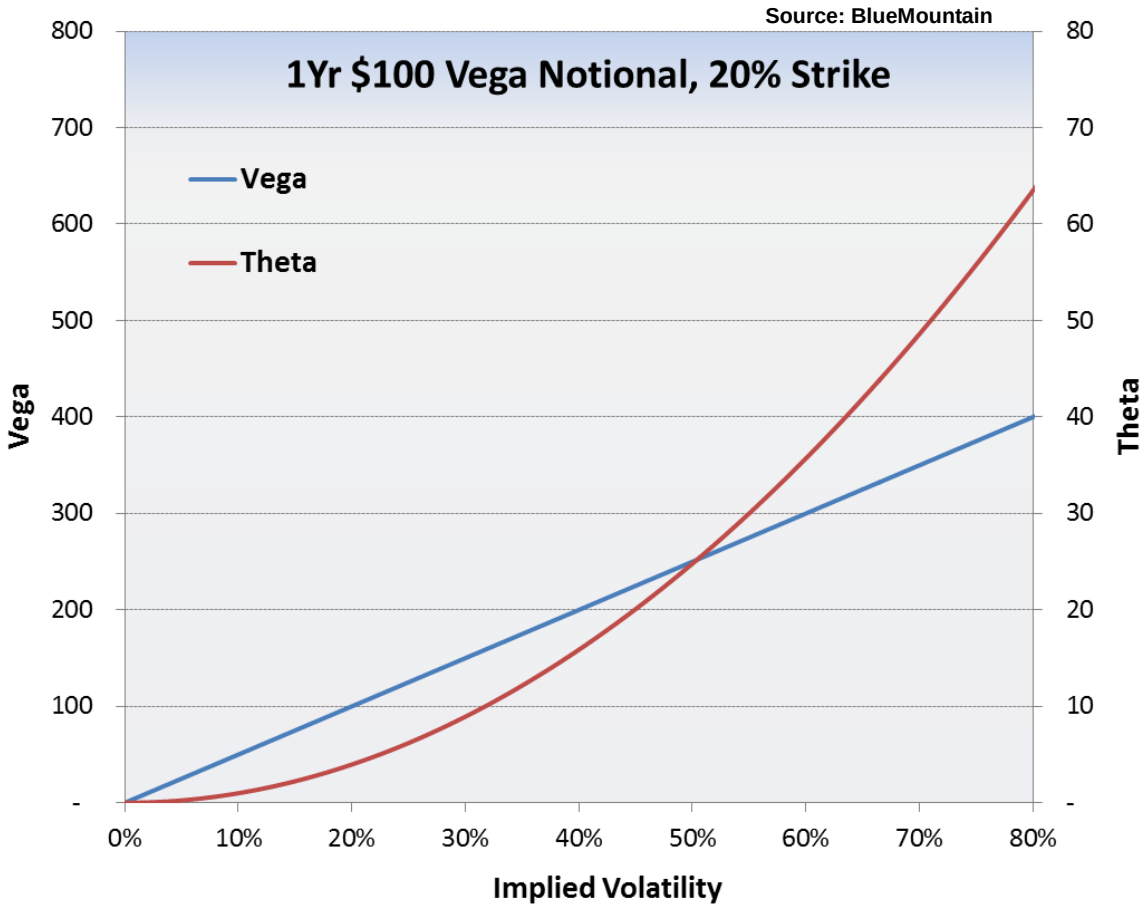
$$\frac{T-t}{T}\frac{K_{t,T}}{K_0}$$

Dollar Gamma

$$\frac{252}{K_0 T}\times 100$$

Daily Theta

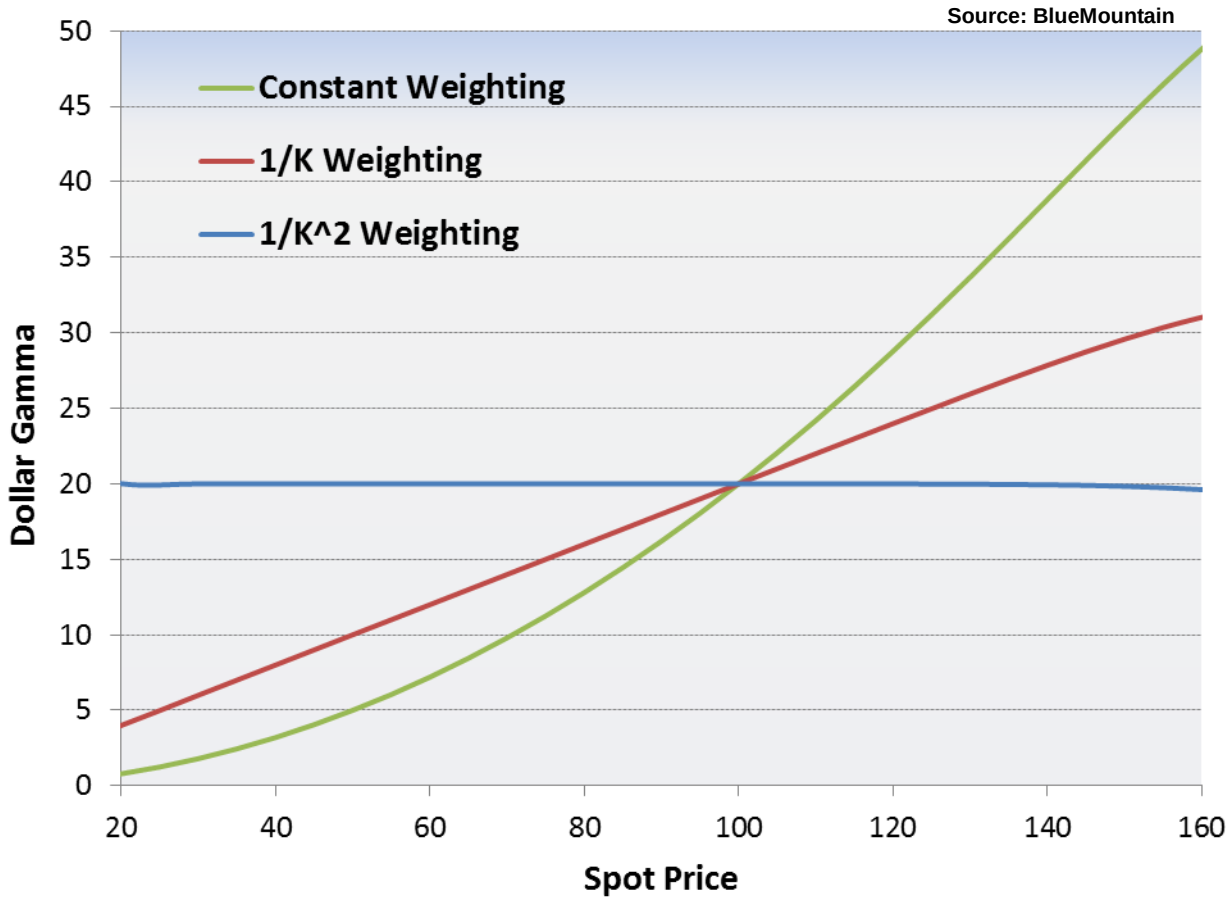
$$\frac{-K_{t,T}^2}{2TK_0}$$



where K_0 is the traded strike, $K_{t,T}$ is the strike of a variance swap on day t expiring on day T

Variance Future Replication through Options

- A defining feature of variance swaps is constant dollar gamma
- A portfolio of options weighted by the inverse square of the strike price achieves constant dollar gamma



Variance Advanced Exposures

Fair Strike	**The variance fair strike is a probability-weighted sum of squared option implied volatilities***
Skew	**Given the typical equity skew (put IV over call IV), the skew delta of the variance swap will be negative**
Convexity	**Convexity has a significant impact on the premium of variance over ATM volatility***

*As indicated by Gatheral (2006)

Trading Variance Instead of Options

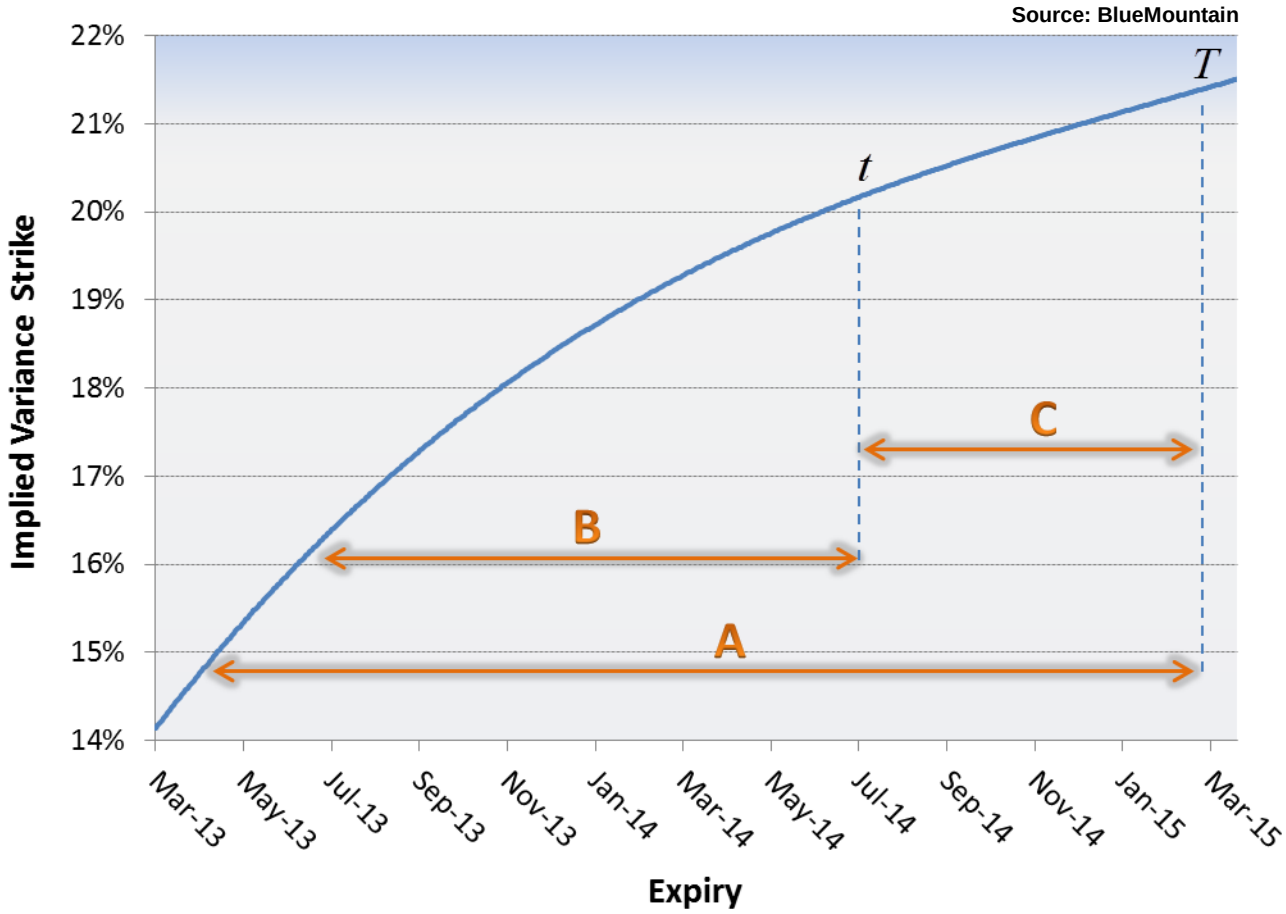
For a pure bet on realized variance, there's nothing better than a variance future (or swap)

- Delta-hedging involves significant transaction costs and time
- Options PnL incorporates substantial path dependency unless you build an entire portfolio
- Liquidity may be difficult for very far OTM options, and they are necessary to maintain constant exposure
- Basis between futures and spot
- Correlation of spot and volatility can alter true gamma
- Trading forward variance requires only two trades instead of hundreds

Forward Variance

Variance is additive linearly over time, so taking exposure to forward variance is simple

$$K_{t,T} = \sqrt{\frac{T}{T-t}K_T^2 - \frac{t}{T-t}K_t^2}$$



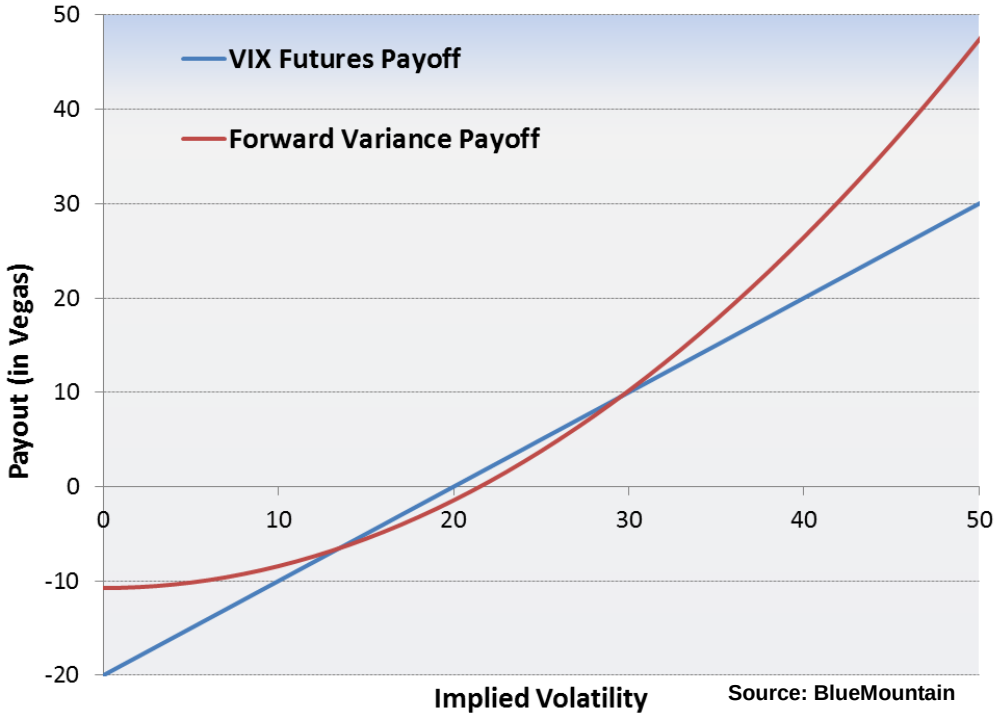
Part 2: Forward Variance v. VIX Futures

Variance versus VIX

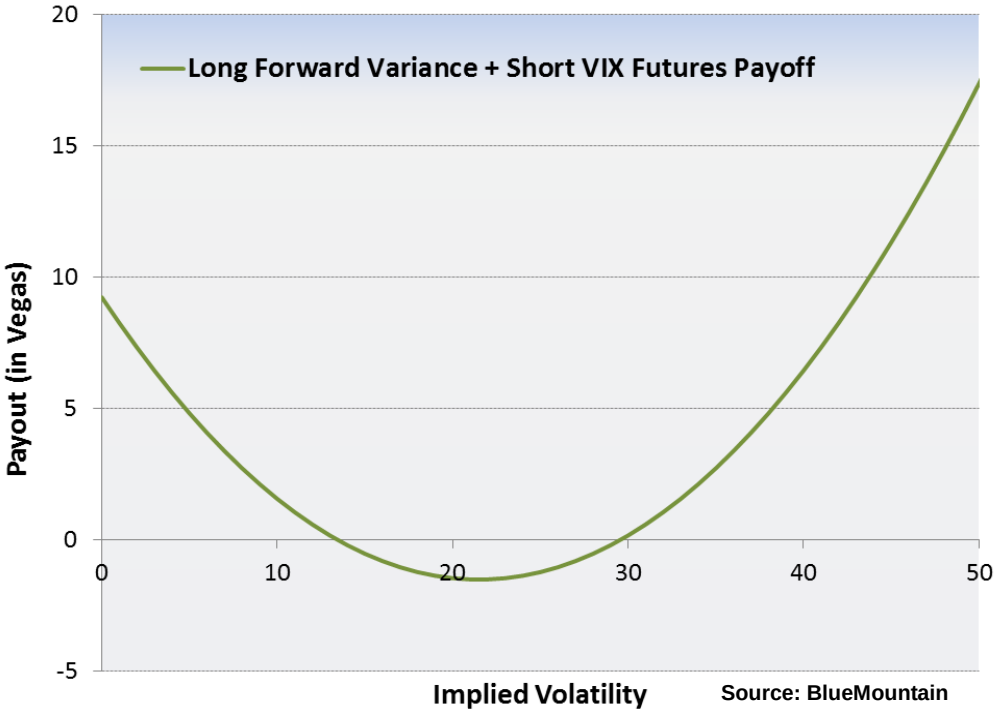
- Due to the convexity of the variance payoff (after adjusting for day-count and different start dates):

$$Fwd\ Var \geq VIX\ \ Fut$$

- In other words, if the strike of the products is the same, there is an arbitrage opportunity
- The fair spread between the two instruments depends on the expected variance of the VIX future



Source: BlueMountain



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How To Value the Convexity Premium

Theoretical

Carr and Wu (2006) illustrate that the difference between the fair strike of the forward variance and the VIX future is simply the risk-neutral variance of the VIX future

$$E(X^2) - (E(X))^2 = \text{var}(X)$$

$$(Fwd\ Var)^2 - (VIX\ \ Fut)^2 = \text{var}(VIX\ \ Fut)$$

- **Where can we find a fair estimate of the variance of a VIX future???**
- **Similar to how one can calculate the fair SPX variance strike from a strip of SPX options, one can also calculate the expected realized variance of a VIX future from VIX options***

$$\text{var}(VIX\ \ Fut) \approx (VIX\ \ Fut)^2 \times \frac{T}{365}\left(\frac{VVIX_T}{100}\right)^2$$

where :

T is calendar days until expiration

$VVIX_T$ refers to a $VVIX$ calculation with expiry T days in the future, not the standard 30 day calculation

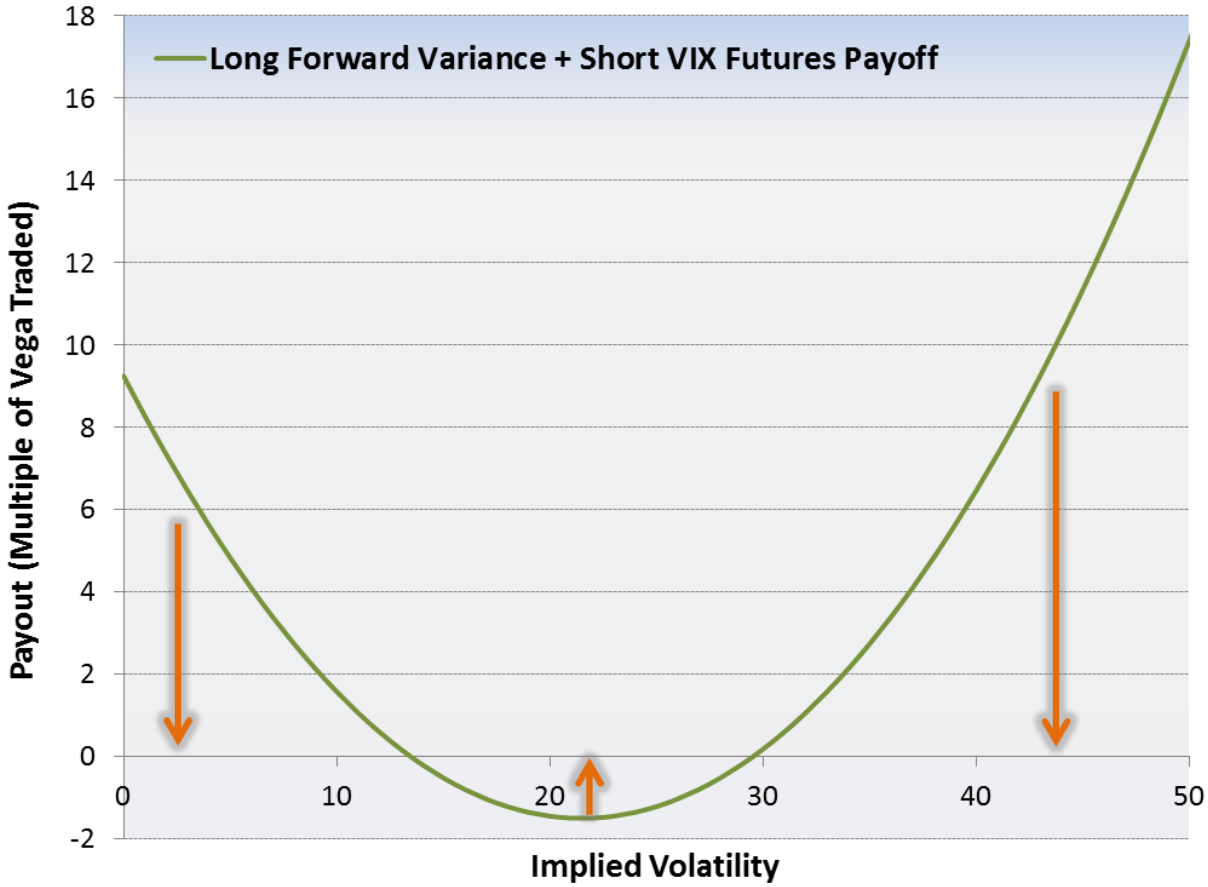
*Translated from CBOE white paper

How To Value the Convexity Premium (Cont.)

Intuitive

Going back to the long variance / short VIX trade, the trade wins on the wings but loses in the middle

Combining short VIX options with that position will push the payoff towards 0 in all states of the world



How To Value the Convexity Premium (Cont.)

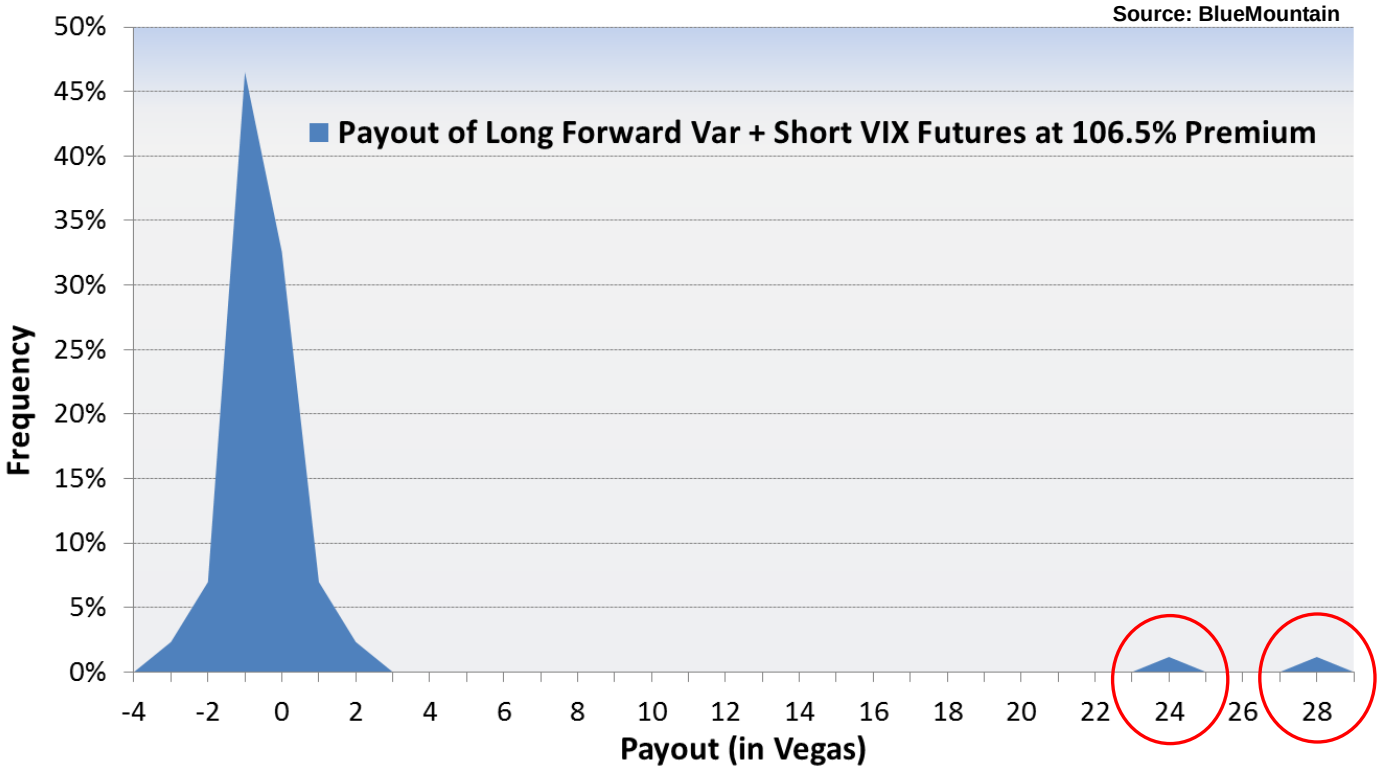
Historical Backtest (2006-Current)

- Solving for a fair ratio of variance over VIX future, 2 months before VIX settlement
- Assumes one-month variance future converges to VIX level on day of VIX settlement

Results:

- Break-even ratio over the entire period is roughly 106.5%
- However, the calculation is greatly skewed by 2 observations in 2008

Source: BlueMountain



Break-Even Ratio	
2006-2007	102.5%
2008-2009	110.0%
2010-Current	105.0%

How To Value the Convexity Premium (Cont.)

Second Historical Backtest (2006-Current)

- Once again looking at a two month period, this backtest assumes vega hedging after each day
- Simplifying assumption is to base the change in variance vega on the move in the VIX future

Results:

- From an overall perspective, the break-even ratio is lower at 104.8%
- Looking closely at each period, hedging makes a substantial difference

Break-Even Ratio

	No Vega Hedging	Daily Vega Hedging
2006-2007	102.5%	103.8%
2008-2009	110.0%	104.9%
2010-Current	105.0%	105.1%

PAYOUT STATISTICS FOR 105% RATIO (In Vegas)

No Vega Hedging

	2006-'07	2008-'09	2010-Current
Mean	(0.4)	1.6	(0.0)
Std Dev	0.4	8.0	0.9
Min	(1.1)	(2.4)	(1.6)
Median	(0.4)	(0.7)	(0.1)
Max	0.9	29.4	2.3
Data Points	24	24	38

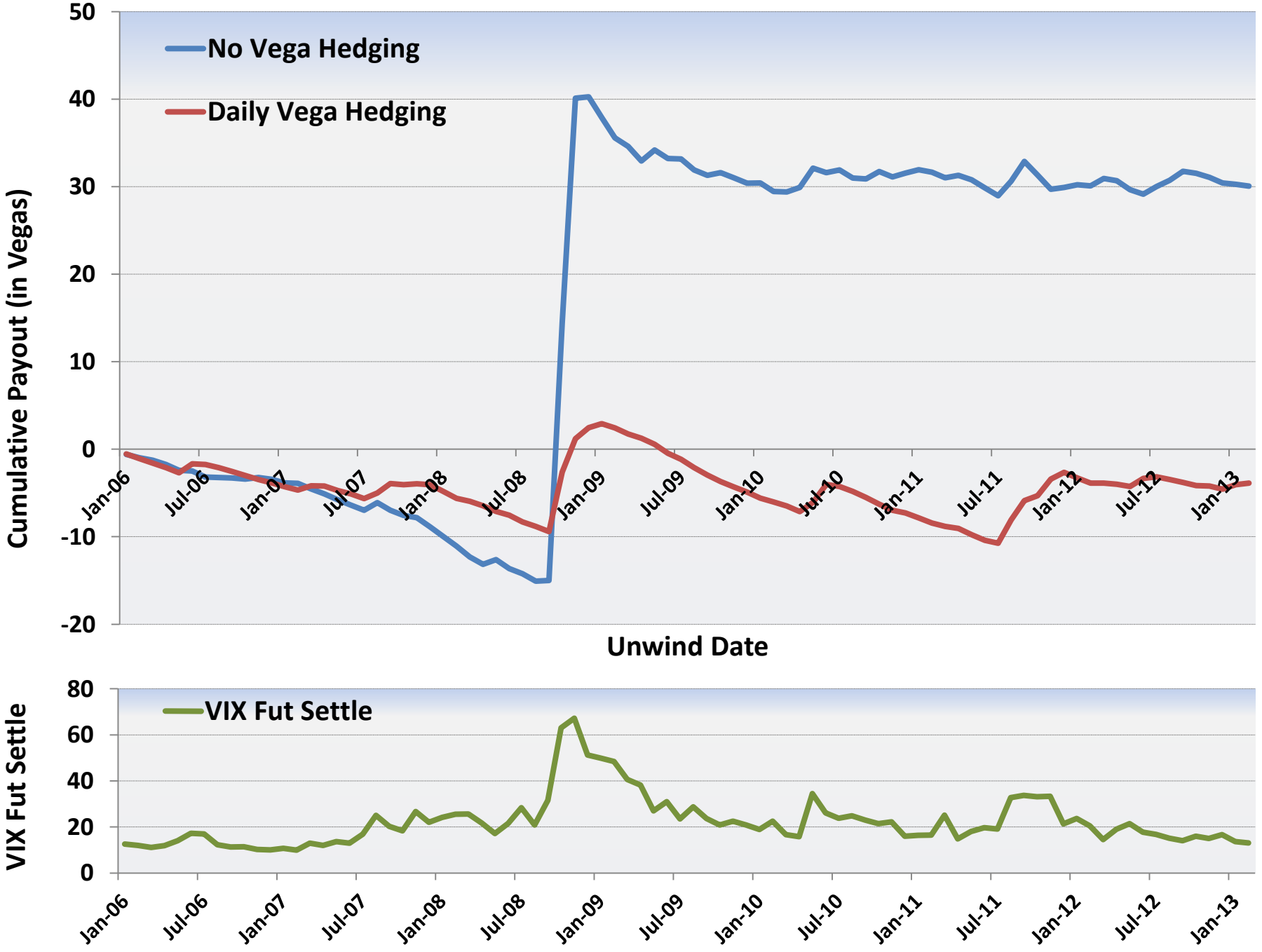
Source: BlueMountain

Daily Vega Hedging

	2006-'07	2008-'09	2010-Current
Mean	(0.2)	(0.0)	0.1
Std Dev	0.5	1.8	0.9
Min	(0.6)	(1.0)	(0.8)
Median	(0.4)	(0.6)	(0.3)
Max	1.1	6.8	2.7
Data Points	24	24	38

Source: BlueMountain

How To Value the Convexity Premium (Cont.)



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