Benn Eifert
@bennpeifert

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Ok so people did get this directionally right. There are several smaller effects that were pointed out (loss of convexity on the straddle as spot moves, etc) but the first-order effect here is “smile delta”. In this example, you will lose money on the rally.

> **Benn Eifert** @bennpeifert · Feb 18, 2020
> You buy a S&P volswap and hedge it with a short ATM straddle (same maturity, same vega). Straddle is Black-Scholes delta neutral. Tomorrow the market rallies 1% and the floating-strike volatility smile stays the same shape (but parallel shifts any way you like). You:
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This position will act mechanically with short delta, regardless of what happens to the vol surface (within reason!) or whether implied volatility under or out performs the skew curve.

A fixed strike option’s implied volatility rides up and down the smile curve as the market moves; a floating strike product like a volatility swap does not.

As the market rallies, the option strikes move to the downside relative to spot, and will trade at increasingly elevated vols relative to the volatility swap, since equity index skew is downward sloping.

This doesn’t depend on any assumptions about sticky delta or sticky strike etc. The fixed strike option implied vol will by *definition* go up relative to floating strike vol as we rally and down as we sell off (unless the smile is flat).

In FX derivatives, market convention is to exchange delta on a trade in an adjusted quantity (“smile delta” or “skew delta”) that accounts for this effect. In equity derivatives, Black-Scholes delta is the standard.

In equity index markets where skew is downward sloping, long options positions will behave as if they are longer than Black-Scholes delta says, so the "smile delta" hedge holds more short delta against them than Black-Scholes. And vice versa.

The adjustment is proportional to the option vega times the slope of the skew curve (which is of course a dynamic thing, so care is warranted in smoothing and data noise)

For sufficiently far out of the money puts, an option's adjusted smile delta may be nearly zero - hence the term "rip delta" among old floor market makers, corresponding only half jokingly to the notion of ripping up the delta ticket after a deep-OTM put trade.

This effect doesn't necessarily imply that you "have to" hedge an option on an adjusted smile delta. If you are trying to replicate or hedge a floating strike position, then it likely makes sense.

But for a short option position in a income strategy, for example, the black-scholes hedge may be volatility dampening .