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Benn Eifert QVR Oct 10, 2022

Quick note on the principle of no-arbitrage in derivatives pricing.

Take a simple forward contract for example. If I sell one, I am obligated to deliver the underlying asset to my counterparty on the maturity date of the contract. That asset could be worth a lot more or a lot less than it is today. But that part doesn't matter to me.

I will borrow cash to buy the underlying asset when I sell the forward. Now I have to pay financing over the life of the trade, but when the futures contract matures, I just deliver the underlying asset, which I already own. I have no risk to its price.

As a result, the forward has to price at a level that is determined by the cost of financing for large. sophisticated arbitrageurs, plus any other costs or income generated by owning the underlying (eg dividends on a stock, or storage costs for a commodity).

Some of these carry costs may be imperfectly predictable - eg there could be dividend surprises - so my forward trade may in fact have some risk to how those costs were priced at trade time versus realized over the life of the trade.

(there might be markets for hedging some of those risks as well! see dividend futures)

This is an important idea. Dealers and arbs don't have to think about what an underlying asset might be worth later in order to price forwards. We just hedge them. Under the risk neutral measure, the drift rate of the stochastic process is given by the cost of carry of the hedge.

Then the textbooks start on options and the practical trouble arises immediately. Black-Scholes is not a model in the sense of anyone suggesting the underlying geometric Brownian motion is a causal explanation of option price dynamics.

The Black & Scholes model used for normalization but not as a model

Option prices observed in the market imply highly dynamic and non-normally-distributed distributions of asset prices. We cannot remove all the risk of an options position by hedging only with the underlying asset the way we (almost) can with a forward contract.

We can hedge the local directional exposure to the underlying asset (or the delta). But all the higher order risks - to realized volatility, implied volatility, spot-vol covariance, volatility of volatility - remain.

we can only synthesize these risks out of options, not the underlying. we can construct tight hedges: get lifted on huge size in 3-month downside skew, and find cheaper 6-month downside skew to buy against it.

so the higher-order risks in option markets trade based on supply and demand, on market participants' need for different payoff profiles and their views on future realized volatility dynamics.

there is no analogue for higher order risks to the change of measure that textbooks spend so much time on, where no-arbitrage principles force the drift of the stochastic process to the risk free rate

there are some looser no-arb constraints - butterflies must have positive prices, etc.

there are no-arb principles relating more complex products like variance swaps to the prices of vanilla options, but the underlying risks in options that are in net demand by clients must ultimately be warehoused in portfolios and managed somewhere, they cannot be neutralized

ultimately that is part of the role of derivatives managers -- to understand dislocations driven by end customer transactions and to warehouse basis risk. banks used to do quite a lot of this, they still do some but they are much more risk constrained compared to a decade ago.