Benn Eifert
@bennpeifert

Benn Eifert QVR Sept 27, 2022

ok. this references the big daddy of all elementary confusions in derivatives. Black-Scholes (and related) models, for which Nobel prizes were won: we do NOT use them as models, we use them as normalizations only, as a convenient change of variables.

> **Un gros grec** @kebabroyal_ · Sep 26
> Replying to @guil_lambert and @bennpeifert
> It's always the same confusion. Using Black-Scholes pricing formula ≠ assuming Black-Scholes dynamics for the underlier. The gamma and theta term are proportional, but they don't cancel unless you assume a GBM dynamic, which nobody ever does.

what do I mean here? A model, as I mean it, is a simplified description of truth, of how the world works. We make some assumptions and draw some logical, mathematical conclusions. A normalization is just a different way of describing the same information.

the theory of gravity is a model; it describes how fast an apple will accelerate as it drops from a tree, perhaps simplifying away certain aspects like wind resistance and how it interacts with the shape of the apple

Black-Scholes, taken literally as a model, starts from the assumption that asset prices follow a random process called a geometric brownian motion (GBM). the only uncertainty in a GBM is the direction of movement of the asset price over each tiny increment of time.

this is analogous to flipping a coin over and over again, and counting up the number of heads minus tails. boring AF game. no one in Vegas will play that, even with cocktail waitresses bringing free drinks.

a GBM's movement over any period of time is drawn from the same constant probability distribution. the volatility of the asset price is known, the level of uncertainty in the world never changes over any time horizon. real financial markets are an explosion of chaotic ambiguity.

the implication of Black-Scholes taken literally as a model is that every option, regardless of strike and maturity, trades at a price consistent with a known, constant and identical volatility level in the famous pricing equation.

all of this is obviously absurd. not in the "well, we know its not quite right, its just a model" kind of way; in a "I award you no points, and may God have mercy on your soul" kind of way.

derivatives traders obsess about volatility surfaces -- undulating patterns in option prices that map strike prices and time to maturity into a level of uncertainty about the future price of an asset

those surfaces fully describe the implied probability distribution of future asset prices, which generally look nothing like the normal distribution consistent with a GBM

so why do derivatives traders talk about implied volatility and the Black-Scholes sensitivities of options (delta- how option prices change as the underlying price moves; gamma, or how delta changes as the underlying price moves; etc?

simple: it provides a convenient normalization of option prices into a common, comparable unit of account, regardless of the underlying price, strike, or time to maturity.

that unit of account is the annualized volatility of the underlying price; the rate of unpredictable change; the standard deviation of the probability distribution of future returns.

"hey, this stock right here has a dec23 50-delta call option trading at $3. this other one has a jun23 50-delta call trading at $0.75!" gives me little useful information.

"the first one is trading at 32% implied volatility and the second one at 16%" gives me a lot more. at a minimum i have some idea that the first one should be about twice as volatile as the second one, perhaps tending to move about 2% and 1% per day on average, respectively

(2% ~= 32% / sqrt(252), because implied volatility is an annualized number, and the standard deviation scales with the square root of time. 252 is the rough number of trading days in a year)

Black-Scholes implied volatilities are much easier to work with than raw option prices. they have comparable economic meaning to each other. they are stationary in the statistical sense (ultimately mean reverting) if compared over time for the same time to maturity

When we use Black-Scholes (or a related method, to handle American options with early exercise) to transform inconvenient prices into convenient implied volatilities, we are just applying a change of variables, not imposing model assumptions. https://en.m.wikipedia.org/wiki/Change_of_variables

Obviously, if we compute a different implied volatility for every strike and maturity, on each day, we are not assuming constant and known volatility! We are respecting the probability distribution implied by market prices.

When we then create models of the dynamics of implied volatility surfaces, describing their shapes and patterns and how they change over time. those models impose structure (much less restrictively than Black-Scholes!) and help us explain and predict option price dynamics

When we use greeks like delta from Black-Scholes, keep in mind that we are treating the implied volatility of any option as a free parameter. **conditional on implied volatility**, the relationship between underlying price and option price holds trivially

delta is not an unconditional forecast of the change in option price for a given change in the underlying price. it is a "true by definition" relationship between spot and option price holding implied vol constant. and analogous for gamma, etc.

that is, all the interesting and meaningful work gets translated into understanding the joint behavior of underlying price and the implied volatility surface, and considering what theoretical or empirical models to apply to that problem.

in sum - we obviously do not live in a world of normal distributions and geometric brownian motion; we use Black-Scholes not as a logical model, but as a market standard for an intuitive normalization of option prices into stationary and economically relevant units.