Understanding VIX futures and options

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Since the Chicago Board Options Exchange (CBOE) introduced futures and, subsequently, options on its Volatility Index, or VIX, traders have asked why the contracts don't necessarily track the underlying in the same way other equity futures track their indexes. Others may wonder why the put-call parity is violated for VIX options. Then, there are the options that trade underwater, the vastly different implied volatilities for each expiration cycle and the question of arbitrage between S&P 500 derivatives and VIX contracts.

Thankfully, all of these questions can be answered with theoretical research on VIX futures and options pricing and, along the way, can offer guidance to some practical applications of these products. Our findings also apply to recently launched VSTOXX index futures and options listed on Eurex.

While the value of the VIX, which is generally accepted as a broad measure of market volatility, is derived from prices of S&P 500 index options, it is not simply a weighted sum of underlying options (unlike other equity indexes like the S&P 500, where the index is a weighted sum of component prices). The options from which VIX is calculated sum up to the square of VIX, not VIX itself. This non-linear transformation means that you cannot just buy or sell a basket of options whose expiration price equals the index. Because of this non-linear component, there is no way to statically replicate the VIX.

Because the underlying VIX is not tradable, the futures on the VIX are not tied by the usual cost of carry relationship that connects other indexes and index futures. To price the futures that have no tradable underlying, we must follow a statistical approach based on various factors: the distribution of the VIX, the strength of the trend, mean-reversion and volatility. In a sense, VIX futures are much like options, having their own set of Greeks.

Close relationship

The first and most obvious attribute of VIX futures is that their options can be priced off the futures using the Black-Scholes futures options formula. Put-call parity holds and is observed in the market, but it is the put-call parity with the futures contract as the underlying, not the VIX index. In-the-money options do not trade in discount to their intrinsic value when calculated off the futures.

The futures chain forms a curve that "connects" the current value of the index with its long-term expectation. To simplify: When the VIX is above its long-term mean, the futures chain will likely be in backwardation, and when the VIX is below its long-term mean, the futures chain will likely be in contango. However, the chain does not need to be strictly decreasing or increasing; it can be completely flat, concave, or convex.

The futures and options tend to anticipate moves. If the VIX has made a sharp move up and traders expect it to come down before expiration, futures will trade below the index, and calls will seem relatively cheap. On the other hand, if VIX is low and traders expect the index to revert to some higher level before expiration, futures will trade above the index, and calls will seem relatively expensive. Theoretical models provide quantitative explanation of these features.

The VIX index and futures are connected by a statistical relationship that depends on how fast the VIX tends to move toward its average level, the volatility of the index and how much time is left until expiration. Near expiration, the futures will be close to the index and move in tandem with it, while the long-term futures reflect the long-term expectation of VIX plus risk premium.

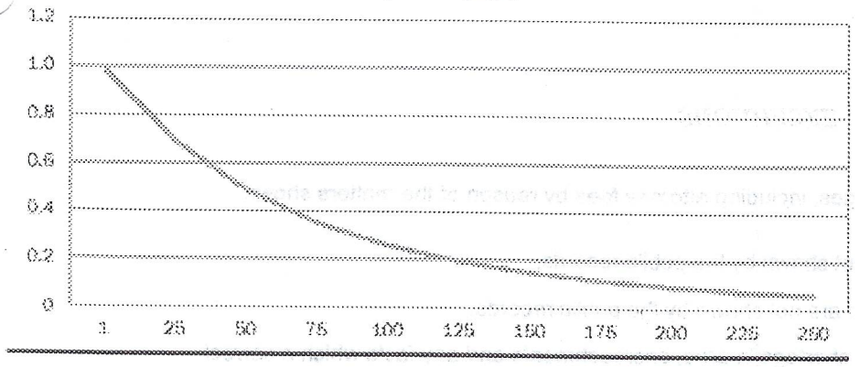
VIX futures have a dynamic relationship with the index, just like options are related to their underlying stock by delta. Traders know that VIX futures do not always follow the index. Jacobs (2009) reports 92% correlation, while Standard & Poor's (2009) reports an 87% correlation. However, because of the dynamic nature of this relationship, the correlation evolves over time and cannot be quantified with a single number.

Close to expiration, the futures will move right in sync with the index (see "Time trials," below). When futures expire in a month (about 25 trading days), the futures will move about half as much as the index. Futures that expire in 125 trading days or more have virtually no correlation to the index. The exact relationship is, of course, dynamic and changes.

TIME TRIALS

This chart shows how sensitive VIX futures are to the index based on time to expiration.

Δ of VIX futures as function of time to expiration (days)



The relationship has other intuitive properties. For example, delta gets smaller as the index increases. This is the result of mean-reversion; futures will move up more when the VIX is below its long-term average and less if it is above it. If the parameters indicate that futures are strongly mean-reverting, delta will be smaller because the current level matters less than its long-term level; if mean-reversion is weak, the delta will be greater because futures will be more sensitive to the current index level.

Knowledge to work

Using the relationship of VIX futures to the index, we can determine how well the contracts will replicate the index. We can calculate the net exposure of different futures and options contracts in terms of the VIX as an aggregate risk measure, or to cross-hedge different futures and options contracts.

Consider, for example, a trader who has multiple positions in VIX options but desires to hedge directional exposure. He does not need to hedge each maturity with its futures contract, but can hedge all of the positions with the most liquid near-term contracts.

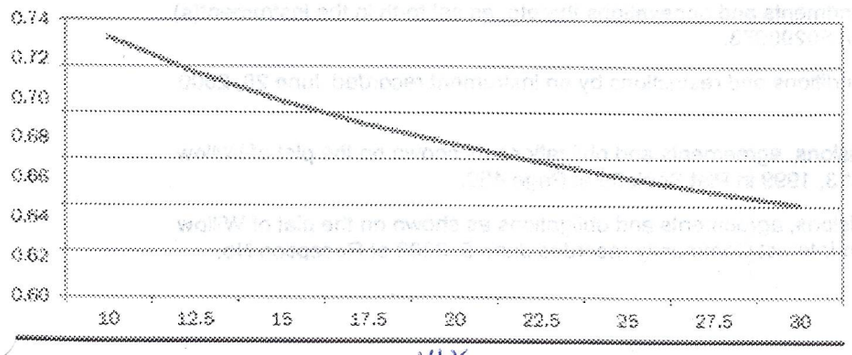
Let's say the trader is short 1,000 delta in May 2011 options. Instead of buying 1,000 May 2011 futures contracts, he can figure out his VIX exposure in terms of near-term November 2010 contracts. If May11 futures delta is 0.5, and Nov10 futures delta is 0.9, the trader has the equivalent of 1,000 * 0.5 = 500 delta of the VIX underlying. In terms of November 2010 futures, it is 500 / 0.9 = 555 delta. So, instead of buying 1,000 May 2011 futures, the trader can buy 555 November 2010 futures, taking advantage of higher liquidity of near-term futures.

These findings also explain why volatility ETNs (VXX, VXZ, XXV in the United States and VIXS, VSXX in Europe) have somewhat low correlation with the VIX itself. For example, the VXX ETN holds the front and second month futures with average maturity of about 30 calendar days and has a return correlation of about 50%, according to our calculation, which is consistent with "Underlying issues" (below).

UNDERLYING ISSUES

The higher the VIX is, the weaker the daily change relationship between the index itself and its futures contract.

Δ of VIX futures as function of VIX index



how far out?
CM?
Γ gamma
Θ Theta

Other Greeks for the futures (Γ, Θ, and vega with respect to volatility of the VIX) can be calculated in a similar manner. It may be surprising to think of Γ for futures, but remember that there is no arbitrage relationship between the index and futures. The only connection is a non-linear statistical relationship that is dynamic in time, which gives non-zero values to Greeks other than delta.

That being said, delta remains by far the most dominant factor, and the relationship between the index and futures can be thought of as "almost" linear.

However, it would be an oversimplification to treat the relationship between the index and futures as completely so. One of the consequences of non-linearity is non-zero expected costs for rolling a futures contract from one month to the next. Intuitively, there is a cost to rolling a long position because there is no riskless arbitrage possible between the two months, so the relationship is purely statistical and the distribution of VIX is positively skewed; there is a price to pay for convexity.

Empirical observations support this. Given the data from the inception of trading in VIX futures until the end of 2009, we estimated the average difference between the front and second month futures on the expiration day to be 0.8 VIX points, or 6.7% of value, and the second month futures is more expensive than the front in 82% of cases. These costs do not include the bid-ask spread; they are statistically and economically significant. These findings are in line with Standard & Poor's (2009), which reports that a portfolio of short-term VIX futures is "expected to suffer from roll loss due to term structure decay" and provide a complementary analysis of rolling costs.

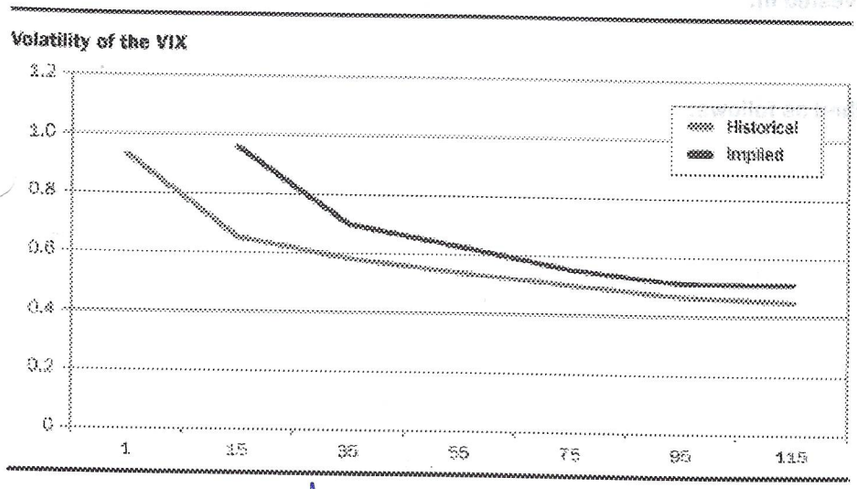
Volatility leak

The implied volatility of options will decrease as time to expiration increases. Because the index is mean-reverting, its volatility will have a term structure with a downward slope. That means if you calculate the annualized standard deviation using one-day returns, then using two-day returns, three-day returns, etc., you will get progressively decreasing numbers.

Using the last 19 years of data from the beginning of 1990 through the end of 2009, we can calculate that one-day volatility of the VIX is 5.9%, or 93% per year, one-month annualized volatility is about 60%, half-year annualized volatility is about 45%, and one-year volatility is only about 35% (see "Volatility through time," below).

VOLATILITY THROUGH TIME

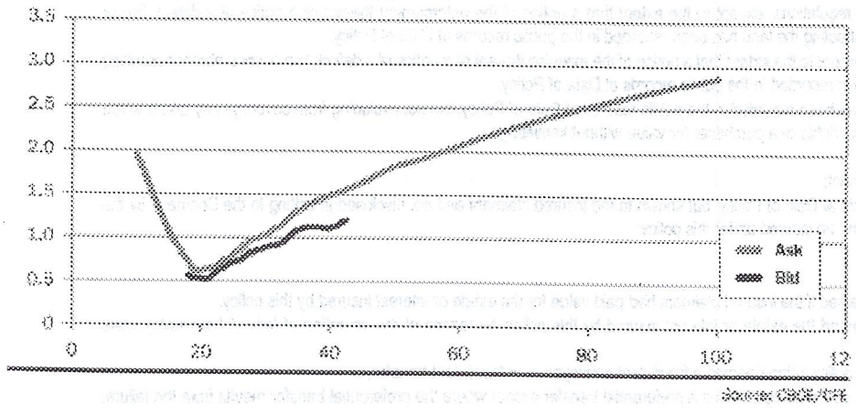
Calculating the annualized standard deviation of VIX, we get progressively smaller numbers the more time we include in the calculation.



ALL SKEWED UP

Using available options strikes 10 through 100, we can calculate the volatility skew in the below chart.

Front Month Volatility Skew



Source: CBOE/CFE

Included on the chart are recent implied volatility levels from at-the-money VIX options (for available maturities). As with other equity indexes, implied volatility is usually higher than historical; however, the general shape and slope of implied and historical volatilities are comparable.

This increase in volatility (as options get closer to expiration) is completely in-the-model behavior caused by mean-reversion and does not correspond to the increase in implied volatilities as normally observed in equity options right before expiration, where it is an out-of-the-model phenomenon.

annualization factor

Theta/Vega hitch

The total theta of VIX options will normally be lower than otherwise expected for other equity indexes. As time approaches expiration, options lose their value; however, implied volatility increases at the same time. The theta of VIX options contains "regular" theta plus an element of vega, because of the term structure of volatility, making time decay smaller. You can empirically confirm that out-of-the-money options remain relatively highly priced, even close to expiration.

VIX traders have pointed out that many trading platforms do not correctly calculate Greeks. At this time, we are not aware of a single commercial platform that provides the correct theta of VIX options.

Credit on call

Because of the term structure of VIX futures, sometimes it would be possible to create calendar spreads for credit — something that would not happen with other equity index options. For example, on Dec. 28, 2009, January 22.5 puts were trading at $1.55-1.70 and February 22.5 puts were trading at $1.10-1.25.

Although this could be thought of as an opportunity for volatility arbitrage, because the total volatility of January options appears to be higher than that of February options, this is actually not the case. The explanation is in a different forward level of the VIX — January futures trading at 21.85 and February futures at 24.65, making January 22.5 puts in the money by 0.65 and February 22.5 puts out of the money by 2.15.

It is not surprising that calendar spreads in VIX options can suffer from unexpected losses. As noted above, the correlation between futures of different maturities can be rather weak, certainly much weaker than other equity indexes. This reason, combined with dynamic term-structure of VIX futures, creates significant risks for calendar spread traders.

It has been widely reported that after spikes in the autumn of 2008, many brokers adopted special risk policies regarding VIX calendar spreads, requiring double margin, or effectively treating different months as if they were different underlyings with their own risk exposures.

Skewed exposure

VIX options have a steep call skew. This is caused by several factors — first, institutional demand to hedge short volatility (and volatility of volatility) exposure creates upward pressure on the calls. The second factor has to do with actual distribution of the VIX. It tends to spike higher much more often than it spikes lower, creating significant hedging risks for call sellers.

For example, on Dec. 30, 2009, the VIX was at 20.30. January 2010 futures had 21 calendar days until expiration, and were trading at 22.70. The strike closest to the money is 22.50 and has implied bid-ask volatility of 61%-68%. The lowest strike for which both implied bid and ask volatilities can be computed is 18, with implied volatility of 57%-72%. The highest strike for which both implied volatilities can be computed is 42.50, with implied volatility of 121%-156%. The skew flattens out for longer-dated options, however the general pattern remains. It is common to see not only mid-market, but also bid implied volatility over 100% in the VIX options

While there is a direct link between the skew in SPX puts and skew in VIX calls, quantifying the exact relationship has been a challenge. VIX skew remains an area of active interest for researchers and for traders it provides the most interesting opportunities.